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Effect of the local acceleration term on the MHD transient free convection flow over a vertical plate

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Abstract

Purpose – The transient hydrodynamics and thermal behavior of free convection flow over an isothermal vertical flat plate is investigated.

Design/methodology/approach – The study focuses on the role of the local acceleration term in the magnetohydrodynamic (MHD) momentum equation. A finite difference method based on a second-order differential equation is used to solve the differential equations.

Findings – It is found that the local acceleration term has insignificant effect on the flow behavior especially at large values of magnetic forces. Also, it is found that the effect of the magnetic forces on the flow hydrodynamics behavior is significant but its effect on the thermal behavior is insignificant. It has been realized that the local acceleration term is usually small compared to the magnetic retarding force, and hence can be neglected.

Research limitations/implications – A quantitative description of the operating and geometrical parameters within which the local acceleration term may be significant is not available in the literature yet. Also, the authors' intention is to improve physical understanding of the hydrodynamic and thermal behaviors of the present problem.

Originality/value – The study provides results concerning the thermal behavior of free convection flow.

Keywords Convection, Flow, Hydrodynamics

Paper type Research paper

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Nomenclature

B	= magnetic flux density
c	= specific heat of fluid at constant pressure
Ec	= modified Eckert number, $\nu^2/x^2c\Delta T$
f	= dimensionless stream function
g	= gravitational body force per unit mass
Gr′	= modified Grashof number, $g\beta\Delta T x^3/4\nu^2$
I	= the parameter, $(g\beta\Delta T/4\nu^2)^{1/4}$
k	= thermal conductivity
N	= modified Hartmann number, $\sigma B^2 x^2/\mu\sqrt{Gr'}$
Pr	= Prandtl number, ν/α
t	= time
$t*$	= dimensionless time, $\sqrt{\mathrm{Gr}'}(\nu t/x^2)$
T	= temperature at any point
T_0	= ambient and initial temperatures
T_w	= wall temperature
u	= axial velocity

v = transverse velocity
x = axial coordinate
y = transverse coordinate

Greek symbols

α = thermal diffusivity, $k/\rho c$
β = volumetric coefficient of thermal expansion
ΔT = the difference $(T_w - T_0)$
η = similarity variable, $Iyx^{-1/4}$
θ = dimensionless temperature, $(T - T_0)/(T_w - T_0)$
μ = dynamic viscosity of fluid
ν = kinematic viscosity of fluid, ν/ρ
ρ = fluid density
σ = the electrical conductivity of the fluid
ψ = stream function, $4\nu Ix^{3/4}f(t,x,\eta)$

Introduction

Magnetohydrodynamic (MHD) convection flow has many important engineering applications in the design of power generators, heat exchangers, pumps, and flow meters, in solving space vehicle propulsion, control, and re-entry problems; in designing communications and radar systems; in creating novel power-generating systems; in developing confinement schemes for controlled fusion; and in nuclear engineering in connection with the cooling of reactors and MHD accelerators.

The problem of MHD natural convection past an infinite or semi-infinite vertical moving plate has been considered by many investigators (Chamkha, 1999; Takhar and Beg, 1997; Vayjravelu and Hadyinicolaou, 1997). Many papers concerned with the problem of MHD steady forced and free convection flow have been published in the literature. As an example, the steady free convection flow of an electrically conducting fluid past over or into different geometries is investigated in the works of Sparrow and Cess (1961), Riley (1964), Raptis and Singh (1983), Vajravelu and Nayfeh (1987), Setayesh and Sahai (1990), Garandet *et al.* (1992), Hossain (1992), Watanabe (1993), Aldoss *et al.* (1995), Al-Nimr (1995), Al-Nimr and Hader (1999) and Aldoss *et al.* (1996). Fewer studies have been carried out to investigate the MHD transient free-convection flow (Sacheti *et al.* 1994; Al-Nimr and Alkam, 1999; Gulab and Mishra, 1977).

The main goal of the present study is to investigate the role of the local acceleration term in the MHD momentum equation and its effect on the hydrodynamics and thermal behavior of the free convection flow over an isothermal vertical flat plate. In the literature about MHD fluid flow, it has been realized that the local acceleration term is usually small compared to the magnetic retarding force, and hence can be neglected. The local acceleration term may be important if an oscillatory driving force is imposed on the system or if the magnetic force is so weak. A quantitative description of the operating and geometrical parameters within which the local acceleration term may be significant is not available in the literature yet. Also, our intention is to improve our physical understanding of the hydrodynamic and thermal behaviors of the present problem.

Governing equations and boundary conditions

We consider unidirectional unsteady laminar free convection flow of an electrically conducting and incompressible viscous fluid over an isothermal vertical plate, immersed in a stagnant fluid of infinite extent and maintained at a constant temperature T_0. The vertical plate is assumed to be nonconducting and the hall effect is negligible. Initially, both wall and fluid have temperature equal to the ambient one T_0 and the imposed magnetic field is absent. Then sudden step change in the wall temperature, from T_0 to T_w, and in the imposed magnetic field, from 0 to B, is applied.

The suddenly imposed magnetic field is assumed uniform and directed toward the positive transverse direction as shown in Figure 1. The impressed electrical field is assumed to be zero and the induced magnetic field of the flow is negligible in comparison with the applied one, which corresponds to a very small magnetic Reynolds number. The fluid is assumed to be Newtonian and obeys the Boussinesq approximation according to which its density is constant except in the gravitational term of the vertical momentum equation. Also, both viscous dissipation and other external sources of heat generation are absent.

Under the above-mentioned assumptions, using boundary layer approximation, the equations of continuity, motion and energy reduce to the following equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = g\beta\Delta T\theta + \nu\,\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2 u}{\rho} \tag{2}$$

$$\frac{\partial \theta}{\partial t} + u\,\frac{\partial \theta}{\partial x} + v\,\frac{\partial \theta}{\partial y} = \alpha\,\frac{\partial^2 \theta}{\partial y^2} - \frac{\sigma B^2 u^2}{\rho c \Delta T} \tag{3}$$

and the initial and boundary conditions are:

$$\begin{aligned} &u(0,x,y) = v(0,x,y) = \theta(0,x,y) = 0 \\ &u(t,x,\infty) = v(t,x,\infty) = \theta(t,x,\infty) = 0 \\ &u(t,0,y) = v(t,0,y) = \theta(t,0,y) = 0 \\ &u(t,x,0) = v(t,x,0) = 0, \quad \theta(t,x,0) = 1 \end{aligned} \tag{4}$$

where x and y are the axial and normal coordinates, u and v are the axial and normal velocities, respectively, g is the gravitational acceleration acting downward in a direction opposite to the x coordinate, $\theta = (T - T_0)/(T_{\mathrm{w}} - T_0)$ the dimensionless temperature, T_0 the ambient temperature outside the boundary layer, T_{w} the wall

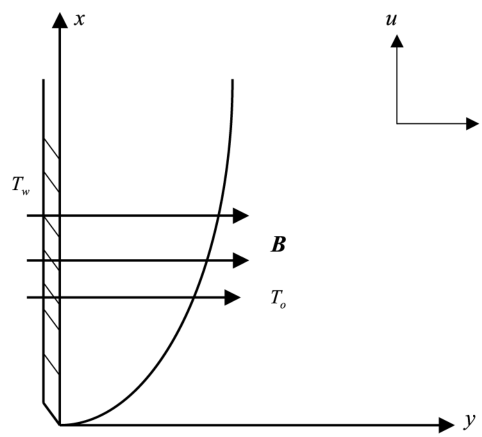

Figure 1.
Schematic diagram of the problem under consideration

temperature, β the thermal expansion coefficient, σ the fluid electrical conductivity, B the magnetic induction, $\Delta T = (T - T_0)$, ρ the fluid density, ν the fluid kinematic viscosity, α the fluid thermal diffusivity and c the fluid specific thermal capacity.

Introducing the stream function ψ, such that $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$ to satisfy the continuity equation, and introducing the following similarity variables

$$\psi = 4\nu I x^{3/4} f(t,\eta), \qquad \eta = I y x^{-1/4}, \qquad I = \left(\frac{g\beta\Delta T}{4\nu^2}\right)^{1/4}$$

with

$$u = 4\nu I^2 x^{1/2} f'(t,\eta), \quad v = \nu I x^{-1/4} f'(t,\eta) - 3\nu I x^{-1/4} f(t,\eta)$$

$$\frac{\partial u}{\partial y} = 4\nu I^3 x^{1/4} f''(t,\eta), \quad \frac{\partial\theta}{\partial y} = I x^{-1/4}\theta', \quad \frac{\partial u}{\partial t} = 4\nu I^2 x^{1/2}\frac{\partial f'(t,\eta)}{\partial t}$$

and the derivative notation in f refers to the derivative with respect to η. Using the above transformations, the governing equations (1)-(4) are reduced to the following similarity equations:

$$\frac{\partial f'}{\partial t^*} + 2(f') - 3ff'' = \theta + f'' - Nf' \tag{5}$$

$$\Pr\frac{\partial\theta}{\partial t^*} - 3\Pr f\theta' = \theta'' + [16N\Pr\mathrm{Ec}\mathrm{Gr}'](f')^2 \tag{6}$$

with

$$\begin{aligned} &f(t^*,0) = 0, \quad f'(t^*,0) = 0, \quad f'(t^*,\infty) = 0, \quad \theta(t^*,0) = 1, \quad \theta(t^*,\infty) = 0,\\ &f'(0,\eta) = 0, \quad \theta(0,\eta) = 0 \end{aligned} \tag{7}$$

where $\Pr = \nu/\alpha$, $N = \sigma B^2 x^2/\mu\sqrt{\mathrm{Gr}'}$, $\mathrm{Ec} = \nu^2/x^2 c\Delta T$, $\mathrm{Gr}' = g\beta\Delta T x^3/4\nu^2$, $t^* = \sqrt{\mathrm{Gr}'}\nu t/x^2$The above system of equations (5)-(7) reduce to the conventional system of equations governing the case of steady free convection problem, once N put to zero, and dropping the unsteady term. This facilitates the using of the computational technique and allows validating the solution against the known solutions. The N-term is nothing but the known Hartmann number divided by the square root of Grashof number. Hartmann number measures the importance of the magnetic force in comparison with that of the viscous force. Also, it is important to notice that the order of magnitude of the magnetic term in the energy equation is of negligible value in comparison with that in the momentum equation especially at low Eckert number. Subsequently, after tracing the hydrodynamics and thermal behavior of the present problem, it is found that the term $[16N\Pr\mathrm{Ec}\mathrm{Gr}'](f')^2$, which represents the ohmic heating, has insignificant effect on the flow behavior under wide range of operating conditions, and as a result, this term will be omitted.

Solution methodology

A finite difference method based on a second-order differential equation (Moran, 1984) is used to solve the differential equations (5) and (6). First, equation (5) which is a third order differential equation is decomposed into a pair of differential equations, one first-order and the other second-order, where

$$u = \frac{\partial f}{\partial \eta} \tag{8}$$

and

$$\frac{\partial^2 u}{\partial \eta^2} - 2u^2 + 3f\,\frac{\partial u}{\partial \eta} - \frac{\partial u}{\partial t^*} + \theta - \mathrm{Nu} = 0 \tag{9}$$

The energy equation (6) is kept as is since it is already a second-order differential equation. A sequential iterative scheme is set up to iterate for the solution of the above three equations (6), (8) and (9).

Before discretizing equation (9), the nonlinear term u^2, is linearized via Newton's method as:

$$u^2 = \bar{u}^2 + 2(u - \bar{u})\bar{u} \tag{10}$$

where $\bar{u}$ considered is known from previous iteration. Equations (6), (8) and (9) are then discretized using the central differencing scheme. This result in a system of the following form:

$$A_i u_{i-1} + B_i u_i + C_i u_{i+1} = D \tag{11}$$

which is a tridiagonal system whose coefficients are known. This system is solved using Thomas algorithm.

A fully implicit scheme is used to introduce the effect of the unsteady term. A system of algebraic equations are solved at each time level. The time marching procedure starts with a given initial field of velocity and temperatures. The system of equation (11) is solved after selecting the time step Δt^*. Once obtaining a converged solution, the solution is assigned old solution and the procedure is repeated to progress the solution by a further time step. Small time steps are needed to ensure accurate results. Number of different values of Δt^* are experimented and $\Delta t^* = 0.001$ is found good enough where the final results attain the same values up to the fifth decimal point.

Boundary layers thicknesses are allowed to grow with each value of MHD parameter N. It is found that the values of $f''(t^*, \infty)$ and $\theta'(t^*, \infty)$ are required to be less than 10^{-5} to assure reaching the edge of the boundary layers. The steady state is said to be reached when the difference between successive values divided by the last value is less than 10^{-5}.

To achieve better accuracy a nonuniform gridding is also used. To check the numerical scheme, the results for the steady case and with zero value of N, are obtained. The results of this special case agree exactly with the known-reported $f''(0)$ and $\theta'(0)$ values.

Results and discussion

After tracing the hydrodynamics and thermal behavior of the present problem, it is found that the term $[16N\mathrm{PrEcGr'}](f')^2$, which represents the ohmic heating, has insignificant effect on the flow behavior under wide range of operating conditions, and as a result, this term will be omitted.

Figure 2 shows the variation of $f'(t^*, \eta)$, which is proportional to the axial velocity u, with the similarity variable η at different modified Hartmann numbers N. As N increases, the flow currents are suppressed and this is justified since the magnetic field has a retarding effect on the velocity. This is also clear from the negative sign associated with the magnetic term in the momentum equation (2). This is true even if the direction of the imposed magnetic field is reversed due to the appearance of B^2 in the equation. Also, as N increases, the location of the maximum velocity is shifted toward the wall side. In free convection, it is known that the temperature obtained from the energy equation affects the flow hydrodynamics behavior and the velocity obtained from the momentum equation affects the flow thermal behavior. However, the effect of temperature on the momentum equation is much larger than the effect of the velocity on the energy equation since temperature is the only source that causes the fluid motion. From Figure 2 it is seen that N has significant effect on the velocity, but due to the insignificant effect of the velocity on the energy equation, the temperature is not sensitive to N.

Figures 3 and 4 show the effect of N on the steady state time (SST), $f''(t^*, 0)$ and $-\theta'(t^*, 0)$ at different Prandtl numbers Pr. The SST is defined as the time required by the fluid and temperature to attain steady state behavior and this parameter is very important since it reflects the importance of the local acceleration term $\partial u/\partial t$. The term $f''(t^*, 0)$ is proportional to the shear stress at the wall and the term $-\theta'(t^*, 0)$ is proportional to the heat transfer at the wall. It is clear from these figures that SST decreases sharply as N increases which indicates that the local acceleration term $\partial u/\partial t$ has insignificant effect on the flow behavior at large values of N and at small values of Pr. This may be justified by returning to equation (2) after neglecting all effects except the local inertial and magnetic terms:

$$\frac{\partial u}{\partial t} + \frac{\sigma B^2 u}{\rho} = 0 \tag{12}$$

Equation (12) assumes the following solution:

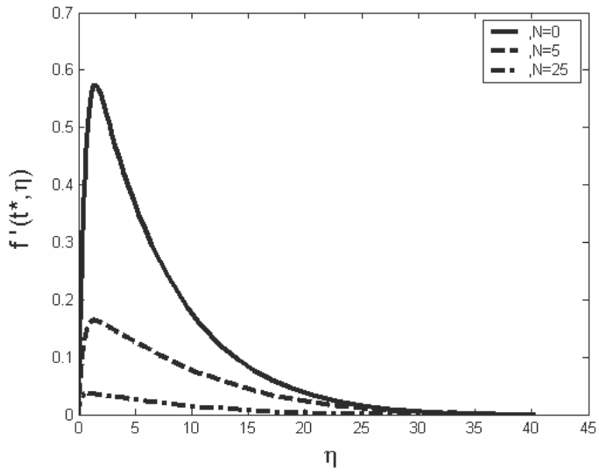

Figure 2.
Effect of the modified Hartmann number N on the axial velocity distribution at Pr = 0.01, Ec = 1.0 and $Gr' = 7 \times 10^{-4}$

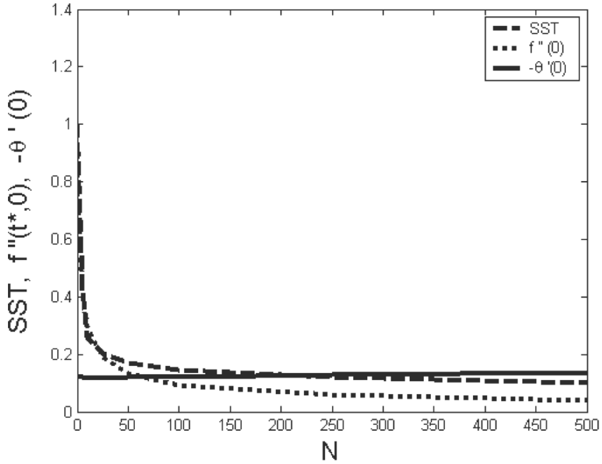

Figure 3.
Effect of the modified Hartmann number N on SST, $f''(t*,0)$ and $-\theta'(t*,0)$ at $\mathrm{Pr} = 0.01$, $\mathrm{Ec} = 1.0$ and $\mathrm{Gr}' = 7 \times 10^{-4}$

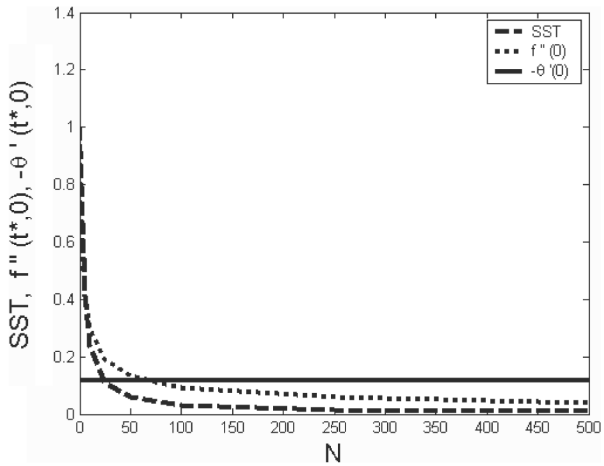

Figure 4.
Effect of the modified Hartmann number N on SST, $f''(t*,0)$ and $-\theta'(t*,0)$ at $\mathrm{Pr} = 0.00001$, $\mathrm{Ec} = 1.0$ and $\mathrm{Gr}' = 7 \times 10^{-4}$

$$u(t) = E\mathrm{e}^{\sigma B^2/\rho}t \tag{13}$$

where E is the constant of integration. Equation (13) shows that as B, or N, increases, the flow attains its steady state behavior in a very short period. Another physical justification relies on the fact that shorter time is required to reach the lower values of the velocities which become lower due to the increase in N. Figures 3 and 4 show that $-\theta'(t^*, 0)$ is not sensitive to the variation in N especially at small Pr. As mentioned previously, $-\theta'(t^*, 0)$ represents the heat transfer which is not sensitive to the variation in N since the energy equation is not strongly affected by the velocity obtained from the momentum equation. The heat transfer through the boundary layer is derived mainly by thermal diffusion which is not affected by the velocity and hence by the magnetic forces. The figures show that $f''(t^*, 0)$ decreases sharply as N increases and its behavior is very close to the SST behavior. The reduction in $f''(t^*, 0)$ is due to the declination in the velocity.

Figure 5 shows the variation of $f''(t^*, 0)$ with η at different N. At low N, and as η increases, $f''(t^*, 0)$ decreases and then increases to reach zero asymptotically and this is a typical and predicted behavior. However, at large values of N, $f''(t^*, 0)$ decreases asymptotically to zero without showing the optimum minimum behavior. The figure shows that the wall shear stress decreases as N increases and this is justified previously.

Figure 6 shows the transient behavior of $\theta'(t^*, 0)$ at different N. Again, the figure shows that the flow attains its steady state behavior very fast and the effect of N on this behavior is insignificant and this is justified previously.

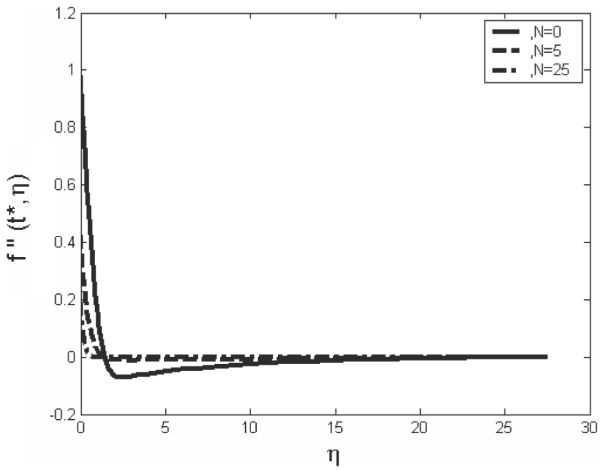

Figure 5.
Effect of the modified Hartmann number N on $f''(t^*,0)$ distribution at $Pr = 0.01$, $Ec = 1.0$ and $Gr' = 7 \times 10^{-4}$

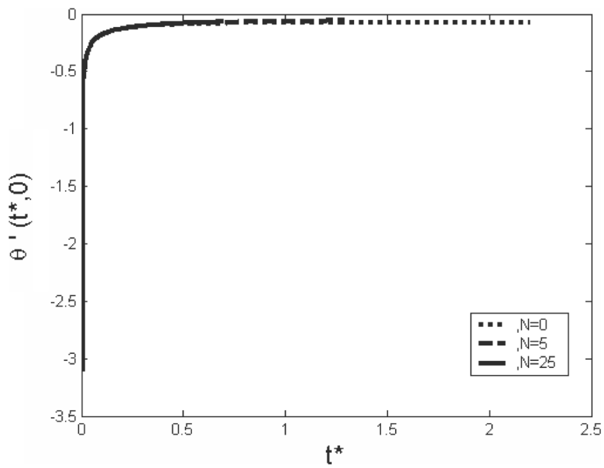

Figure 6.
Effect of the modified Hartmann number N on $\theta'(t^*,0)$ transient distribution at $Pr = 0.01$, $Ec = 1.0$ and $Gr' = 7 \times 10^{-4}$

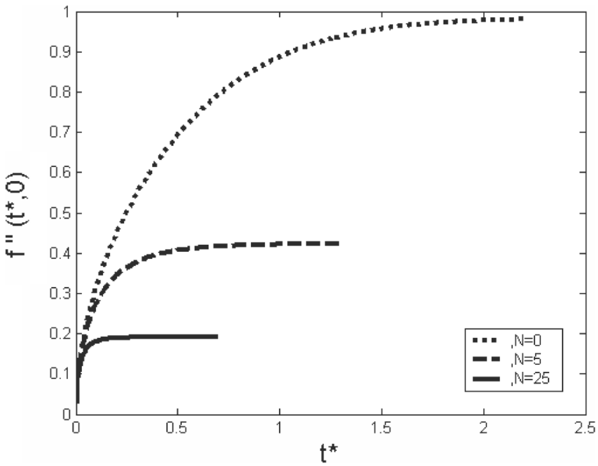

Figure 7.
Effect of the modified Hartmann number N on $f''(t^*,0)$ transient distribution at $\mathrm{Pr} = 0.01$, $\mathrm{Ec} = 1.0$ and $\mathrm{Gr}' = 7 \times 10^{-4}$

Figure 7 shows the transient wall shear stress at different N. As N increases the wall shear stress decreases and attains its steady state values in shorter time. This behavior is also justified previously.

Conclusions

The transient hydrodynamics and thermal behavior of free convection flow over isothermal vertical plate is investigated using similarity approach. The study focuses on the role of the local acceleration term in the MHD momentum equation. It is concluded that the SST decreases sharply as N increases which indicates that the local acceleration term $\partial u/\partial t$ has insignificant effect on the flow behavior at large values of N and at small values of Pr. The effect of the magnetic forces on the flow hydrodynamics behavior is found to be significant, but this is not the case with the thermal behavior. The temperature distribution within the thermal boundary layer is found to be almost linear and the ohmic heating term is found to have insignificant effect on the flow behavior under wide ranges of operating conditions. As N increases, the location of the maximum velocity is shifted toward the wall side. It is concluded that the heat transfer at the wall is not sensitive to the variation in N especially at small Pr while the wall shear stress decreases sharply as N increases.

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